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# MODAL TESTING OF A SIMPLIFIED WIND TURBINE BLADE

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## ABSTRACT

*This paper examines the modal analysis techniques applied in experiments using a uniform and a stepped beam. These simplified shapes are representative of the a wind turbine blade. Natural frequencies have been identified, therefore designers can ensure those natural frequencies will not be close to the frequency of the main excitation forces (1P or $N_b$P with Nb being the number of rotor blades) in order to avoid resonance. The turbine blade is approximated by a cantilever, therefore, it is fully constrained where attached to a turbine shaft/hub. Flap-wise, edge-wise and torsional natural frequencies are calculated. The results found have been compared to numerical results and the exact solution of an Euler-Bernoulli beam. Concurrence is found for the frequency range of interest. Although, some discrepancies exist at higher frequencies (above 500 Hz), finite element analysis proves to be reliable for calculating natural frequencies.*

**KEYWORDS:** *Modal testing, wind turbine, natural frequencies, finite element analysis, Euler Bernoulli beam*

## I. THEORY OF EXPERIMENTAL MODAL ANALYSIS

Modal analysis provides information on the dynamic characteristics of structural elements at resonance, and thus helps in understanding their detailed dynamic behaviour [1]. Modal analysis can be accomplished through experimental techniques. It is the most common method for characterising the dynamic properties of a mechanical system. The modal parameters are:

- The modal frequency;
- the damping factor and,
- the mode shape.

The free dynamic response of the wind turbine blade can be reduced to these discrete set of modes. It should be noted that determination of the damping properties is usually considered to be somewhat uncertain, which relates to the small quantities of the damping characteristics [2].

Relevant prior works are those which acquire wind turbine modal data and those which use the modal data to validate a model. Molenaar [3] performed an experimental modal analysis of a wind turbine with accelerometers distributed over the rotor blades. The natural frequencies of the test were used for comparison with a state-space model of the same turbine. The natural frequencies were used to validate the model parameters of the wind turbine. Griffith et al. [4] have presented modal test results for two series of wind turbine blades tested at Sandia National Laboratories with a specific aim of characterizing the blade structural dynamics properties for model validation purposes. Further information on the tests or the mode shapes can be found in the test report [5].

Real structures have an infinite number of degrees of freedom (DOFs) and an infinite number of modes. They can be sampled spatially at as many DOFs as is desired from a testing point of view. There is no limit to the number of unique DOFs between which FRF (frequency response function) measurements can be made. However, time and cost constraints result in only a small subset of the FRFs being measured on a structure. From this small subset of FRFs, the modes that are within the frequency range of the measurements can be accurately defined [2]. The more the surface of the structure is spatially sampled by taking more measurements, the more definition is given to its mode shapes.

Because a wind turbine blade is generally a large structure (length >20m) with shape and sizes changing along its length it is necessary to treat it in successive cross-sections. The modal analysis of the wind turbine blade is performed by exciting it at a fixed point during the test. This excitation represents the input signal to the system. The output signal consists of accelerations measured at various cross sections along the blade. A finite number of degrees of freedom are used to describe blade motion. The mode shapes of the blade are assumed to be described by deflection in the flap-wise and edge-wise directions as well as by rotation of the chord about the pitch axis (torsion). The rigid body motion can be described by three DOFs in each cross-section. Two flap-wise DOFs describe the flap-wise deflection and torsion (denoted $U_y$ and $\theta_t$) and one edge-wise DOF describes the edge-wise deflection (denoted $U_x$). The rigid body motion (response) can be derived as a function of the three amplitudes of the DOFs in the following form [2]:

$$U = Ax \qquad (1)$$

where $U$ is the motion of the cross-section and $x$ (excitation) is the corresponding amplitudes in the three DOFs of the cross-section.

$$U = \begin{Bmatrix} U_x \\ U_y \\ \theta_t \end{Bmatrix} \text{ and } x = \begin{Bmatrix} x_i \\ x_{i+1} \\ x_{i+2} \end{Bmatrix}$$

and $A$ (the FRFs) is a three by three matrix given by the positions of the three DOFs.

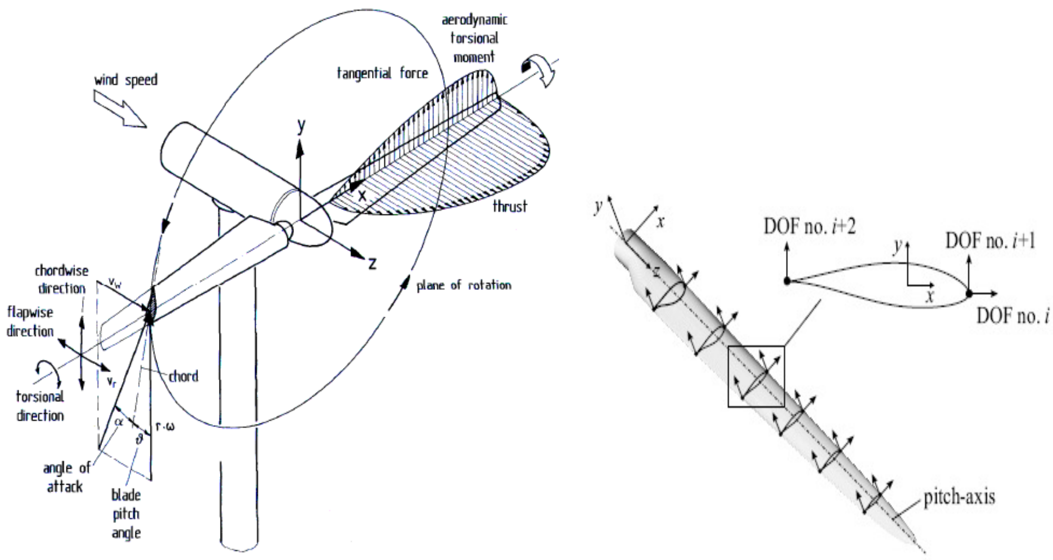


**Figure1:** The degrees of freedom for a wind turbine blade. (Adapted from [2] and [8])

Using Eq. (1) a mode shape of the blade can be estimated in a number of cross-sections, presuming the corresponding modal amplitudes *(U* and $x$ *)* have been measured in the three DOFs of each cross-section.

The rest of the paper is organised as follows: in section 2, the extraction of modal properties is described. In section 3 the experimental setup and modal testing is presented while the equipment used are described. In section 4 and 5, the results found using an experimental modal analysis of a uniform beam and a stepped beam are respectively presented and discussed thoroughly. Finally in section 6 the concluding remarks are presented.

# II. EXTRACTION OF MODAL PROPERTIES

## 2.1. Modal properties from an eigenvalue problem

To introduce this mathematical concept the linear equation of free motion for the blade is considered. The motion of the blade is described by *N* DOFs as shown in Figure 1. The deflection in DOF $i$ is denoted $x_i$, and the vector $x$ describes the discretized motion of the blade. Assuming small deflections and moderate rotation of the blade cross-sections, the linear equation of motion can be written as [2]:

$$M\ddot{x} + C\dot{x} + Sx = 0, \qquad (2)$$

where dots denote derivates with respect to time, and the matrices $M$, $C$ and $S$ are the mass, damping and stiffness matrices. Inserting the solution $x = ve^{\lambda t}$ into Eq.(2) yields

$$(\lambda^2 M + \lambda C + S)v = 0, \quad (3)$$

which is an eigenvalue problem. The solution to this problem is the eigenvalues $\lambda_k$ and the corresponding eigenvectors $v_k$ for *k* = 1, 2, . . ., *N*. The eigenvalues of a damped blade are complex and given by:

$$\lambda_k = \sigma_k + i\omega_k \qquad (4)$$

where $\sigma_k$ and $\omega_k$ are respectively the damping factor and the modal frequency for mode $k$. The relationship between natural frequencies ($f_k$), logarithmic decrements ($\delta_k$) and the eigenvalues are:

$$f_k = \frac{\omega_k}{2\pi} \text{ and } \delta_k = -\sigma_k / f_k \qquad (5)$$

The natural frequencies and logarithmic decrements are obtained from the eigenvalues, and mode shapes are obtained from the eigenvectors. The above equations indicates that the problem of determining natural frequencies, logarithmic decrements, and mode shapes of a blade could be solved if one had a way to measure mass, damping, and stiffness matrices. Such measurements are, however, impossible. Instead one can measure transfer functions in the frequency domain which hold enough information to extract the modal properties [2].

## 2.2. From transfer functions to modal properties

A transfer function describes in the frequency domain the response in one DOF due to a unity forcing function in another DOF. It is defined as [2]:

$$H_{ij}(\omega) \equiv X_i(\omega) / F_j(\omega) \qquad (6)$$

where,
$\omega$ is the frequency of excitation
$X_i(\omega)$ is the Fourier transform of the response $x_i(t)$ in DOF $i$
$F_j(\omega)$ is the Fourier transform of a force $f_j(t)$ acting in DOF number *j*.

By measuring the response $x_i$ and the forcing function $f_j$, then performing the Fourier transformations, the transfer function $H_{ij}$ can be calculated from Eq.(6). This transfer function is one of *N* × *N* transfer functions which can be measured for blade with *N* DOFs. The complete set of functions is referred to as the transfer matrix $H$. To understand this basic principle of modal analysis, consider the linear equation of motion (Eq.(2)) for the blade with external excitation.

$$M\ddot{x} + C\dot{x} + Sx = f(t) \qquad (7)$$

Where the vector *f* is a forcing vector containing the external forces $f_j(t)$ which may be acting in the DOFs $j$ = 1, 2, . . ., *N*.
The transfer matrix can be derived as [2]:

$$H(\omega)=\sum_{k=1}^{N}H_k(\omega)=\sum_{k=1}^{N}\frac{v_k v_k^T}{(i\omega-\sigma_k-i\omega_k)(i\omega-\sigma_k+i\omega_k)} \qquad (8)$$

This relation is the basis of modal analysis. It relates the measurable transfer functions to the modal properties $\omega_k$, $\sigma_k$, and $v_k$. Each mode $k$ contributes a modal transfer matrix $H_k$ to the complete transfer matrix. Hence, a measured transfer function can be approximated by a sum of modal transfer functions [2]:

$$H_{ij}(\omega)\approx\sum_{k=1}^{N}H_{k,ij}(\omega), \qquad (9)$$

where the modal transfer functions $H_{k,ij}(\omega)$ by decomposition can be written as [2]

$$H_{k,ij}(\omega)=\frac{r_{k,ij}}{i\omega-p_k}+\frac{\overline{r_{k,ij}}}{i\omega-\overline{p_k}} \qquad (10)$$

where the bar denotes the complex conjugate. $p_k=\sigma_k+i\omega_k$ is called the pole of mode $k$ and $r_{k,ij}=v_{k,i}v_{k,j}$ is called the residue of mode $k$ at DOF $i$ with reference to DOF $j$. Thus, a pole is a complex quantity describing the natural frequency and damping of the mode. A residue is a complex quantity describing the product of two complex modal amplitudes. The modal properties are extracted from measured transfer functions by curve fitting functions derived from Eq. (9) and Eq.(10), with poles and residues as fitting parameters.

The purpose of the present study is to perform modal analysis and identify flap-wise and edge-wise natural frequencies. Comparison will be made between those experimental results and numerical results (their detailed description is available in [7]. Therefore, the simplified blade shapes shown in Figure 3 and Figure 4 were chosen.

# III. METHODS

## 3.1. Experimental setup

There are several methods available to measure the frequency response functions needed to perform a modal analysis. The most important differences between these methods are in the number of inputs and outputs and in the excitation method used:

- The single input single output (SISO) methods and,
- the multiple input multiple output (MIMO) methods.

The two most common excitation methods are:

- Excitation using an impact hammer and,
- excitation using an electrodynamical shaker.

Each of these methods has specific advantages and disadvantages which determine the most suitable measurement in a specific case. The advantages and disadvantages of each method are discussed by Ewins [1].

In order to measure the frequency response functions of the turbine blade model a single input, single output impact test with fixed boundary conditions is performed. The reasons behind the choice for this type of test are:

- The purpose is only to extract the natural frequencies;
- all the test equipment needed for an impact test were readily available making an impact test cheaper than alternative methods for which most of the equipment required is not available and,
- the extra sensors and data processing capability needed to implement an alternative testing method were also unavailable.

## 3.2. Exciting modes with impact testing

Impact testing is a quick, convenient way of finding the modes. Impact testing is shown in Figure 2. The equipment required to perform an impact test in one direction are:

- An ***impact hammer*** with a load cell attached to its head to measure the input force.

- An ***accelerometer*** to measure the response acceleration at a fixed point and in a fixed direction.
- A two channel ***FFT analyser*** to compute frequency response function (FRFs).
- ***Post-processing modal software*** for identifying modal parameters and displaying the mode shapes in animation.

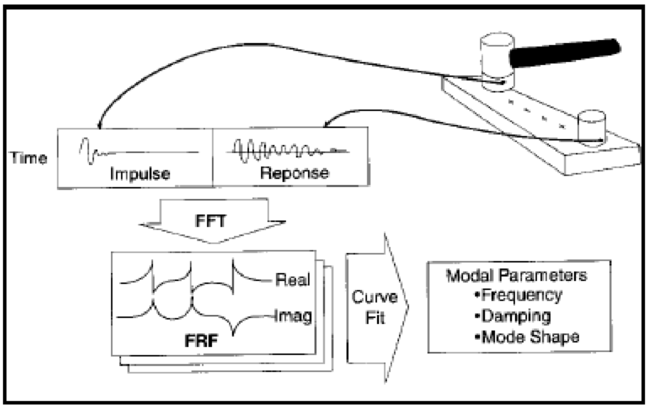


**Figure 2:** Impact testing. (Adapted from [9])

The idea of exciting a structure with an impact hammer is actually simple:

- One strikes a structure at a particular location and in a particular direction with an impact hammer. The uniform and stepped beams are successively excited in flap-wise direction;
- the force transducer in the tip of the impact hammer measures the force used to excite the structure;
- responses are measured by means of accelerometers mounted successively at the tip of the uniform and stepped beams;
- the force input and corresponding responses are then used to compute the FRFs (frequency response functions) and,
- desktop or laptop computer with suitable software collects the data, estimates the modal parameters and displays results.

Experimental modal analysis has been performed successively on uniform and stepped beams, to extract natural frequencies. The uniform beam was chosen as a starting point because the analytical solution is available [8]. The stepped beam is an approximation for a tapered wind turbine blade. A wind turbine blade can be seen as beam of finite length with airofoil profiles as cross sections. A rectangular cross section representing a cross section of the blade can give qualitatively appropriate results in a simpler way.

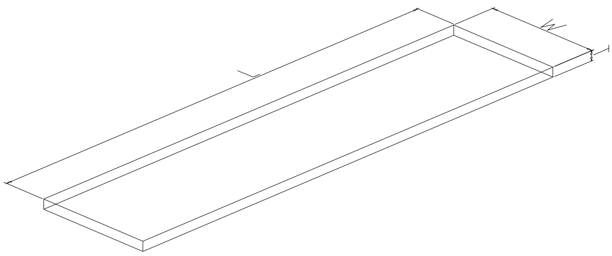

**Figure 3:** Dimensions of the uniform beam used in the experiment.

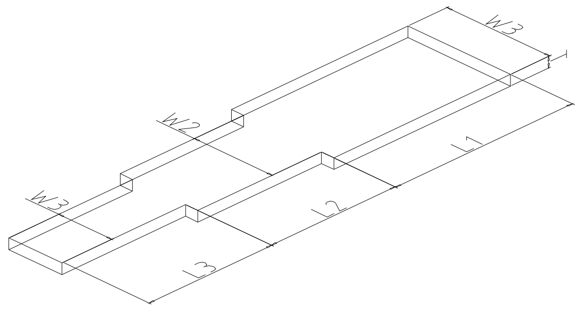


**Figure. 4:** Dimensions of the uniform and stepped beam used in the experiment.

Modal testing has been performed in order to extract the natural frequencies of the test beam. The following paragraphs are a brief description of the set-up, necessary equipment and procedure for performing the test

(1) Test Beam
A test beam is fastened to a table with a clamp at one location. Clamping details are shown in Figure 5.

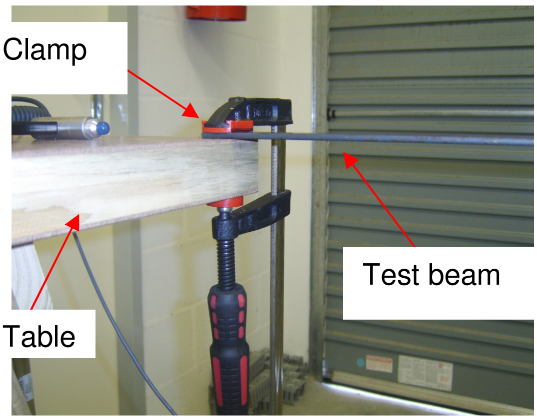


**Figure 5:** Clamping details.

(2) Impact Hammer
Model 086C02 from PCB Piezotronics is used to cause an impact. It consists of an integral ICP quartz force sensor mounted on the striking end of the hammerhead. The hammer range is about $\pm 440\ N$. Its resonant frequency is near $22\ kHz$. Figure 6 shows the hammer and the beam.

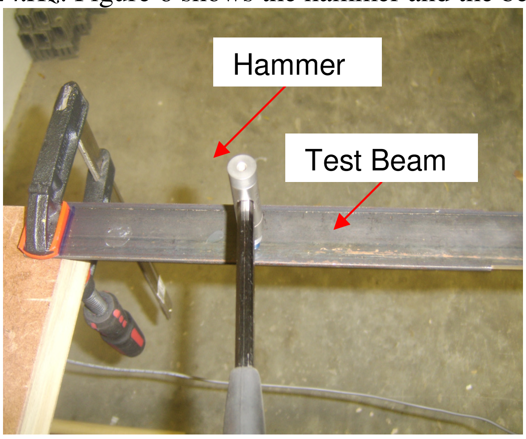


**Figure 6:** Impact hammer.

(3) Accelerometer

IEPE Accelerometer, Model IA11T, from IDEAS SOLUTION is used in the test. It is capable of measuring frequencies from *0.32 Hz* to *10 kHz* and voltage sensitivity is $10.2\ mV/(m/s^2)$.

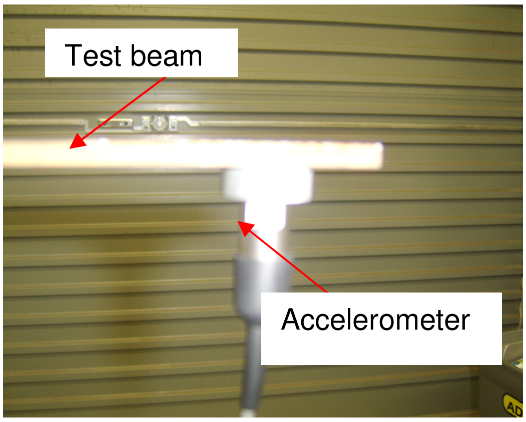


**Figure 7:** Accelerometer on the beam.

(4) Dynamic Signal Analyser
Measurement of the force and acceleration signals is performed using a "OneproD MVP-2C" 2-channel dynamic signal analyser. It samples the voltage signals emanating from the impact hammer and accelerometer. The sensitivity information of the sensors is used to convert the voltages to equivalent force and acceleration values. The dynamic signal analyser also performs the transformations and calculations necessary to convert the two measured time domain signals into a frequency response function. Measurement data may be processed on a computer using Vib-Graph software.

**Figure 8:** Dynamic signal analyser

For this work Vib-Graph was used. From the experimental data, it determines the dynamic parameters of a system. Three additional methods are used for obtaining the natural frequencies:
- Exact solution of Euler-Bernoulli beam equations;
- MATLAB program for one-dimensional finite element models and,
- NX5 three dimensional models.

## IV. Experimental Modal Analysis Results and Discussion for a Uniform Beam

The uniform beam had a rectangular cross-section with width *W* and thickness *T*. The length of the beam was *L*. The values of these dimensions are shown in Table 1.

**Table 1:** Material and geometric properties of the uniform beam

| Geometric properties | | | Material properties (mild steel) [10] | | |
|---|---|---|---|---|---|
| $L(mm)$ | $W(mm)$ | $T(mm)$ | $E(mN/mm^2)$ | $\rho\,(kg/mm^3)$ | $\nu$ |
| 795 | 40 | 4.45 | $206\times10^{6}$ | $7.85\times10^{-6}$ | 0.3 |
| $L$: length<br>$W$: width<br>$T$: thickness | | | $E$: Young's modulus<br>$\rho$ : density<br>$\nu$ : Poisson's ratio | | |

The performed modal analysis gives estimates of only flap-wise natural frequencies. The results are based on the measurements performed on uniform beam as described in section 3. Figure 9 shows a screenshot of Vib-Graph after measured transfer functions are imported. Crosses (**+**) indicate natural frequencies. The natural frequencies, obtained from the modal analysis, are presented in Table 2.

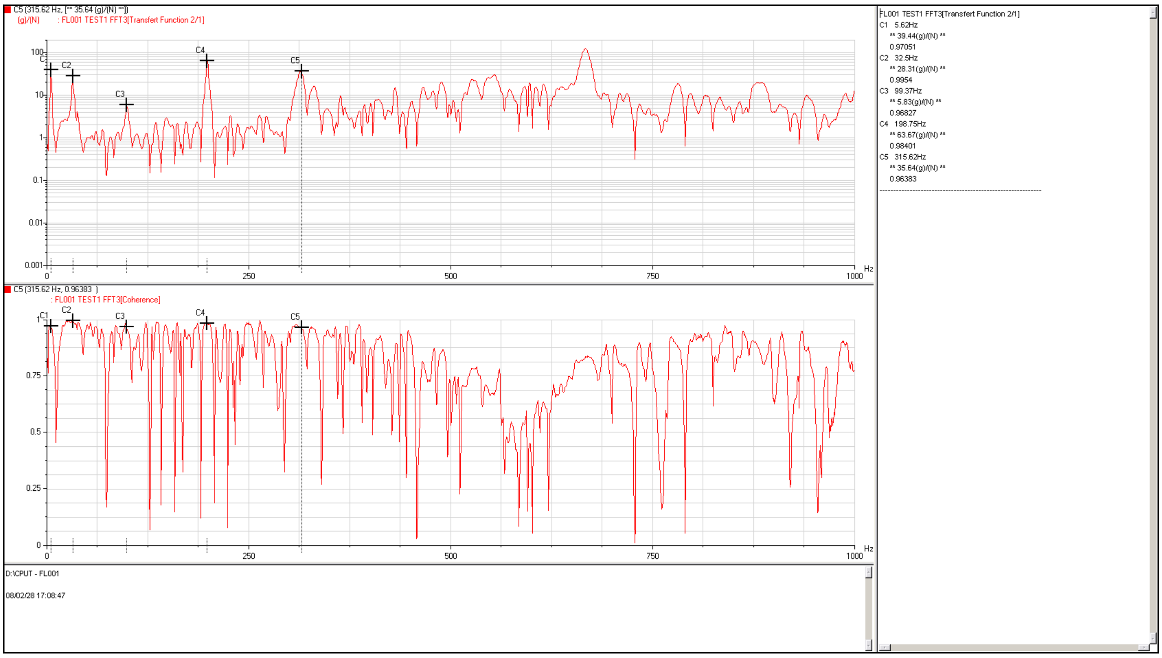


**Figure 9:** Measured transfer functions imported into Vib-Graph

The results found using the four different methods have been compared. It should be noted that:

- The experimental modal analysis provides only the first five flap-wise natural frequencies and,
- The MATLAB program provides only flap-wise and edge-wise natural frequencies (their detailed description is available in [7]).

Therefore, the comparison has been limited to the data available.
The exact solution of natural frequencies of the beam can be obtained as follow [11]:

$$f = \frac{\beta^2}{2\pi}\sqrt{\frac{EI}{\rho A}} = \frac{(\beta L)^2}{2\pi}\sqrt{\frac{EI}{\rho A L^4}} \qquad (11)$$

With values of $\beta$ in Eq. (11) determined from:

$$\beta_1 L = 1.875104$$

$$\beta_2 L = 4.6940914$$

$$\beta_3 L = 7.8547577$$

$\beta_4 L = 10.995541$

$\beta_5 L = 14.137168$

$A$ and $I$ represents respectively the cross-section and the area moment of inertia.

The frequencies of torsional modes of a rectangular cantilever with a width to thickness ratio greater than six may be approximated by [11]:

$$f_n = \frac{(2n-1)}{4L}\frac{2T}{W}\sqrt{\frac{G}{\rho}} \tag{11}$$

Where the shear modulus $G$ is given by:

$$G = \frac{E}{2(1+v)} \tag{12}$$

**Table 2:** Measured and computed natural frequencies

| | Exact solution [Hz] | Measured frequencies [Hz] | Computed frequencies [Hz] | |
|---|---|---|---|---|
| | | | MATLAB | NX5 |
| Flap-wise | 5.83 | 5.62 | 5.890 | 5.918 |
| | 36.5 | 32.5 | 36.92 | 37.08 |
| | 102 | 99.3 | 103.5 | 103.8 |
| | 200 | 198.75 | 202.8 | 203.5 |
| | 331 | 315.62 | 335.3 | 336.6 |
| Edge-wise | 52.4 | | 52.52 | 52.34 |
| | 328 | | 327.9 | 324.2 |
| | 919 | | 919.9 | 891.6 |
| | 1800 | | 1802 | 1704 |
| | 2977 | | 2981 | 2734 |
| Torsional | 224.79 | | | 219.4 |
| | 449.57 | | | 659.1 |
| | 674.36 | | | 1102 |
| | 899.14 | | | 1549 |
| | 1123.93 | | | 2005 |

Some conclusions can be drawn from the previous table:

- There are no significant discrepancies between the exact solution and MATLAB results;
- highest edge-wise frequencies introduce some discrepancies between MATLAB and NX5 results (their detailed description is available in [7]). This may be due to the limitation of the one dimensional model (MATLAB) compared to the three dimensional model (NX5) when it comes to computing higher natural frequencies;
- highest torsional frequencies also produce some discrepancies between exact solution and NX5 results for similar reason as above. Interestingly, Larsen et al. [2] in their study compares the results from the modal analysis with the corresponding results from the finite element analysis. Better agreement has been found for the deflection components associated with low natural frequencies than for deflection components associated with higher natural frequencies. The same tendency was also observed in the estimation of natural frequencies. The bending torsion coupling has been identified as a reason for those discrepancies. It has been found that these deflections are difficult to resolve experimentally (due to small signal levels) as well as numerically (due to lack of sufficiently detailed information on the material properties). The numerical model is seen to over-estimate the structural couplings. Although, torsional natural frequencies are not included in experimental results, this may also explain discrepancies at higher frequencies.

- the closeness between the experimental (for at least the first five flap-wise and edge-wise and the first torsional) results and the finite element analysis results means that finite element analysis can be used as a good computational tool and,
- the closeness between the analytical results, the measured frequencies and the computed frequencies means that natural frequencies can be predicted accurately by any of those methods.

## V. EXPERIMENTAL MODAL ANALYSIS RESULTS AND DISCUSSION FOR A STEPPED BEAM

The stepped beam (Figure 4) had a rectangular cross-section with widths *W*1, *W*2, *W*3 and thickness *T*. The length of each portion was given by *L*1, *L*2, *L*3. The values of these dimensions are shown in Table 3.

**Table 3:** Material and geometric properties of the stepped beam

| | Geometric properties | | | Material properties [10] | | |
|---|---|---|---|---|---|---|
| | $L(mm)$ | $W(mm)$ | $T(mm)$ | $E(mN/mm^2)$ | $\rho\,(kg/mm^3)$ | $\nu$ |
| Portion1 | 295 | 40 | 4.5 | $206\times10^6$ | $7.85\times10^{-6}$ | 0.3 |
| Portion2 | 250 | 36 | 4.5 | $206\times10^6$ | $7.85\times10^{-6}$ | 0.3 |
| Portion3 | 250 | 30 | 4.5 | $206\times10^6$ | $7.85\times10^{-6}$ | 0.3 |
| *L*: length<br>*W*: width<br>*T*: thickness | | | | *E*: Young's modulus<br>$\rho$ : Density<br>$\nu$ : Poisson's ratio | | |

Hereafter the MATLAB, NX5 (their detailed description is available in [7]) and experimental modal analysis results are presented. No exact solution is available for the stepped beam.
The performed modal analysis gives estimates of only flap-wise natural frequencies. The results are based the measurements performed on the stepped beam described in section 3. Fig. 6 shows a screenshot of Vib-Graph after measured transfer functions are imported. Crosses (+) indicates natural frequencies. The natural frequencies, obtained from the modal analysis, are presented in Table 4.

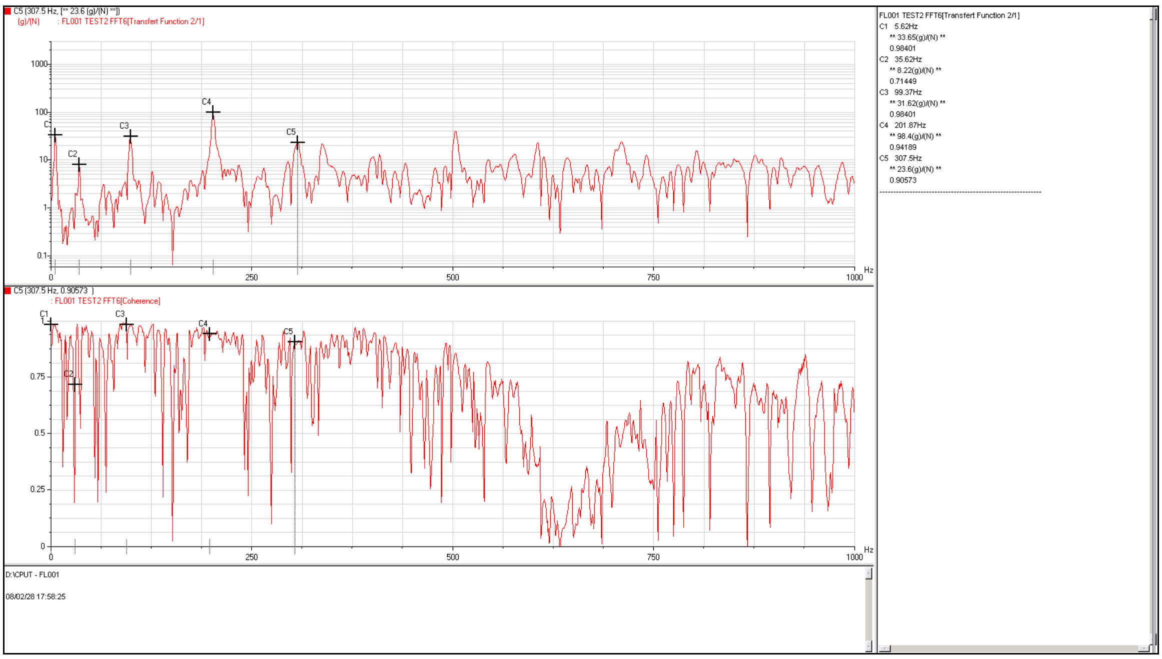

**Figure 10:** Measured transfer functions imported into Vib-Graph

The results found previously have been compared. This comparison has been limited to the data available.

**Table 4:** Measured and computed natural frequencies

| | Measured frequencies [Hz] | Computed frequencies [Hz] | |
|---|---|---|---|
| | | MATLAB | NX5 |
| Flap-wise | 5.62 | 6.61 | 6.636 |
| | 35.62 | 37.88 | 38.02 |
| | 99.37 | 103.6 | 103.9 |
| | 201.87 | 202.7 | 203.5 |
| | 307.5 | 335.3 | 336.2 |
| Edge-wise | | 57.62 | 57.55 |
| | | 305.4 | 301.9 |
| | | 807 | 786 |
| | | 1587 | 1516 |
| | | 2629 | 2447 |

It can be seen that the measured frequencies results and the computed frequencies remain close. However, as previously, some discrepancies can be observed for the highest frequencies. Interestingly, Jaworski and Dowell [12] in their study predicted the three lowest natural frequencies of a multiple-stepped beam using:

- A classic Rayleigh–Ritz formulation;
- commercial finite element code ANSYS and,
- experimental results from impact testing data.

It has been shown that:

- Classical Rayleigh–Ritz provides more accurate results at the highest frequency for global parameters once sufficient degrees-of-freedom are introduced and,
- the disagreement between beam model and experimental results is attributed to non-beam effects present in the higher-dimensional elasticity models, but absent in Euler–Bernoulli and Timoshenko beam theories. This conclusion is corroborated by predictions from one-, two-, and three-dimensional finite element models.

It should be specified, however, that this study is not concerned with higher natural frequencies.

## VI. CONCLUSIONS

In this study the natural frequencies of three different beams have been investigated:

- A uniform beam (Figure 3);
- a stepped beam (Figure 4) and,

Four different methods are used for obtaining the natural frequencies:

- Exact solution of Euler-Bernoulli beam equations;
- MATLAB program for one-dimensional finite element models;
- NX5 three dimensional models and,
- experimental modal analysis.

To validate results, the outputs from different methods are evaluated and compared.

The following conclusions have been drawn:

- good agreement between experimental analysis, NX5 and MATLAB results has been confirmed for the frequency range of interest. Therefore both NX5 and the MATLAB program can be use to calculate natural frequencies for any other isotropic material. This means that an effective method to compute natural frequencies of a simplified wind turbine blade was developed;
- some discrepancies between measured frequencies results and the computed frequencies can be observed for highest frequencies;
- the range between 0.5 Hz and 30 Hz is of relevance to wind turbine blades. Higher flap-wise natural frequencies, all edge-wise and all torsional natural frequencies are out of this range of concern for this model (Table 3).
- modal testing should definetely be performed to extract the flap-wise natural frequencies, which are more likely to coincide with excitation frequencies.

## ACKNOWLEDGEMENT

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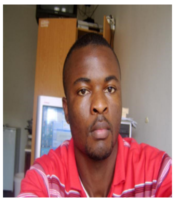

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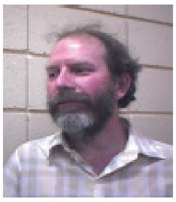

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